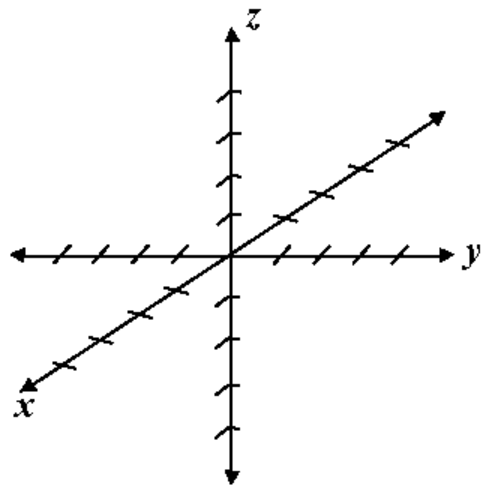
Chapter 9 Vectors in Space

9.1 Rectangular Coordinates in Space



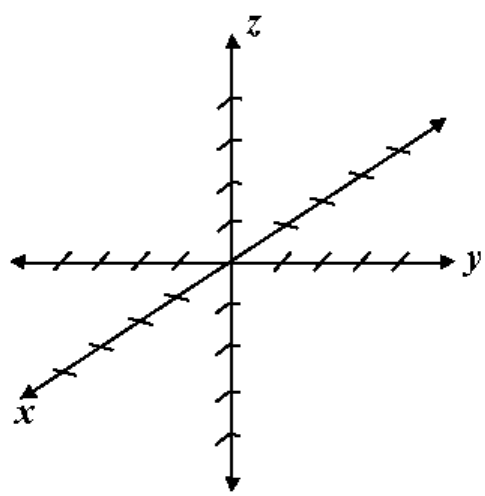
Ordered triples: (x, y, z) = (-5, 2, 4)

Distance formula:

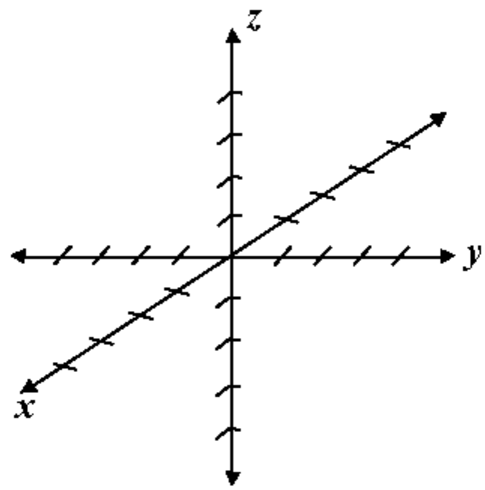
$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Formula for a sphere:
$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$

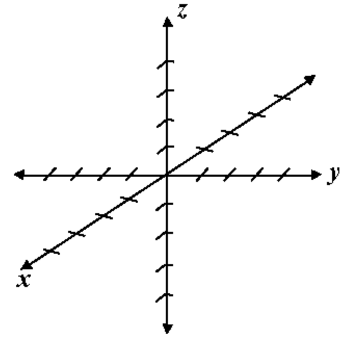
EX 1: The given points are the legs of a triangle in space. Plot the points and use the distance formula to name the triangle as accurately as possible. (right? isosceles?)
A (0, 1, -1), B (1, 1, 2), C (3, 4, -2)



EX 2: Find the equation of a sphere with a center at (2, 3, -3) and tangent to the x-y plane



EX 3: Given points A (2, 0, -1)and B (3, 1, -2), find the equation that describes the set of all points equidistant from A and B.



9.2 Vectors in Space

The same properties of vectors that we had in 2D are true for 3D

a)The letter **k** is used to represent the unit vector in the z direction.

b)If **u** = a**i** + b**j** + c**k** then|**u**| = $\sqrt{a^2+b^2+c^2}$

c)Let **u** = a**i** + b**j** + c**k** and **v** = d**i** + e**j** + f**k,** then the dot product of **u** and **v** is ad + be + cf

d)If **u** and **v** are orthogonal, then the value of their dot product is 0

e). $\cos\theta = \dfrac{ad+be+cf}{|\vec{u}||\vec{v}|} = \dfrac{\vec{u}\circ\vec{v}}{|\vec{u}||\vec{v}|}$

EX 4: Find the angle between vectors **a** and **b** if

a = 3**i** + 2**j** + 2**k**

b = **i** – 2**j** – **k**

EX 5a: Given vectors

a = **i** + **j**

b = **j** + **k**

c = **i**+ **k**

v = 2**i** + 3**j**

Express **v** as a linear combination of **a**, **b** and **c**

EX 5b: Given vectors $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

$\mathbf{b} = \mathbf{i} - \mathbf{j} - \mathbf{k}$

$\mathbf{c} = \mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$

$\mathbf{v} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$

Use the fact that the vectors **a**, **b** and **c** are orthogonal in pairs to express **v** as a linear combination of **a**, **b** and **c**

EX 6: Given vectors $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$

$\mathbf{b} = \mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$

$\mathbf{v} = 18\mathbf{i} - 13\mathbf{j} + 16\mathbf{k}$

Find the vector **c** such that $\mathbf{v} = 2\mathbf{a} - \mathbf{b} + 5\mathbf{c}$

9.3 Lines in Space

Our goal: To develop the equation of a line in vector form in space. To do this we take two steps with the links below.

Step 1: develop the equation of a line in 2D in vector form
Mathematica development of 2D vector equation of a line

Step 2: extend this idea to space (3D)
3D vector equation of a line

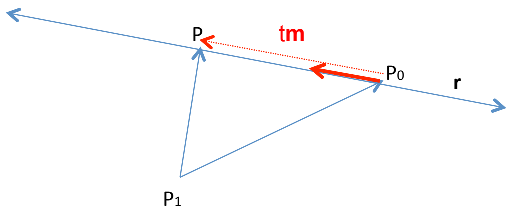
http://www.ies.co.jp/math/java/vector/chok3D/chok3D.html

Step 3: See what this looks like in space (3D) and discuss axis intercepts
Picture of 3D line with vector equation

http://www.walter-fendt.de/m14e/line3d.htm

EX 7: Find a vector equation of the line that contains points (1, -2, 4) and (2, 1, 3)

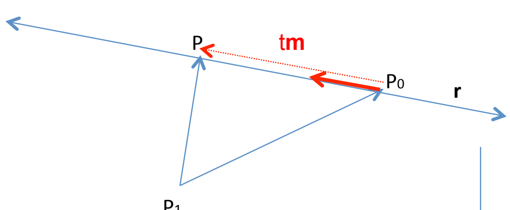
EX 8: Find the distance from P_1 (2, 2, 1) to **r** = < 3 + t, 4 – 2t, 1 – t >



We will make use of two facts:

1. $\overrightarrow{P_1P} = \overrightarrow{P_1P_0} + t\mathbf{m}$ (vector addition)
2. $\overrightarrow{P_1P} \circ \mathbf{m} = 0$ (they are perpendicular)

EX 8: Find the distance from P_1 (2, 2, 1) to **r** = < 3 + t, 4 – 2t, 1 – t >



We will make use of two facts:

1. $\overrightarrow{P_1P} = \overrightarrow{P_1P_0} + t\mathbf{m}$ (vector addition)
2. $\overrightarrow{P_1P} \circ \mathbf{m} = 0$ (they are perpendicular)

Step 1: break up **r** into its components

Step 2: use vector addition to write
$\overrightarrow{P_1P} = \overrightarrow{P_1P_0} + t\mathbf{m}$

Step 3: use the dot product to find t ($P_1P \perp \mathbf{m}$)
$\overrightarrow{P_1P} \circ \mathbf{m} = 0$

Step 4: sub this specific t into $\overrightarrow{P_1P}$

Step 5: find $|\overrightarrow{P_1P}|$

Practice at your desk:

1. Find a vector equation of the line that contains the point $P_0(2, -1, 3)$ and is parallel to $< 1, -2, 4 >$

2. Find the distance from P_1 (1, -2, 0) to $\mathbf{r} = < 2 - t, 1 - t, 1 + t >$

9.4 Planes in Space

A link to a website that shows the same notations as in our text:

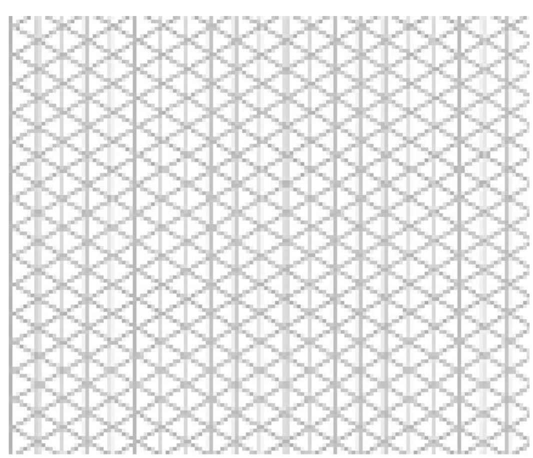
Four handy things to remember from this:

- The equation for any plane can be written as $ax + by + cz = d$ (scalar equation) OR
- The equation for any plane can be written as $\mathbf{n} \circ (\mathbf{r} - \mathbf{r}_0) = 0$ (vector equation)
- The values of a, b, and c in the above equation supply us with the coefficients of the normal vector
- Any line perpendicular to the plane is parallel to the normal vector

EX 9:

a) Find the equation of the plane through P_0 (1, 1, 1) that is perpendicular to $\mathbf{r} = < -1 + 2t, t, 3t >$

b) Draw the first-octant part of the plane

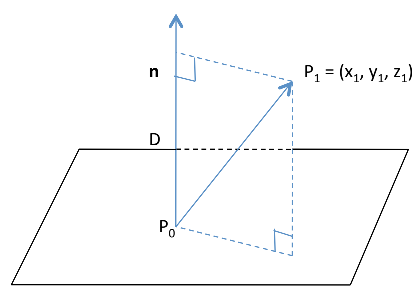


EX 10: Find a vector equation of the line that contains point (2, 0, 1) and is perpendicular to the plane x – 2y + 3z +5 = 0

EX 11: Find the distance from point (2, 0, 1) to the plane x – 2y + 3z +5 = 0

If we let $P_1 = (x_1, y_1, z_1)$ and the plane = ax + by + cz + d = 0 then we can use the following formula:

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|\overrightarrow{P_0P_1} \circ \vec{n}|}{\|\vec{n}\|}$$



EX 12: Find a vector equation of the plane that contains points (1, 2, 3), (0, 1, 2), and (1, 0, 1)

EX 13a: Find the point where the line $\mathbf{r} = \langle 2t, 2 - t, 1 \rangle$ intersects the plane
Q: $x + y - 3z = 1$

EX 13b: Find the point where the line $\mathbf{r} = \langle 3 - 2t, 1 + t, 3t \rangle$ intersects the plane
Q: $x - y + z = 0$