In this Stewart's Chapter 8, we introduced the dot product of two vectors and continue in this Chapter with vectors space.

We'll learn about cross product (or vector product) that we'll use to solve problem in three-space.

Cross Product (Vector Product)

Given that $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$, denoted $\mathbf{u} \times \mathbf{v}$ is given by

$$\mathbf{u} \times \mathbf{v} = \left(\begin{bmatrix} u_2 & u_3 \\ v_2 & v_3 \end{bmatrix}, \begin{bmatrix} u_3 & u_1 \\ v_3 & v_1 \end{bmatrix}, \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix} \right)$$

The preceding formula can be remember as the expansion of:

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

Example 1a

Find the cross product of $\mathbf{u} = (3, -2, 0)$ and $\mathbf{v} = (4, 1, 0)$.

Example 1b

Find a vector that is perpendicular to both vectors $\mathbf{u} = (3, -2, 0)$ and $\mathbf{v} = (4, 1, 0)$.

Theorem 1

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors let r and s be scalars.

1. $\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$
2. $(r\mathbf{u}) \times (s\mathbf{v}) = rs(\mathbf{u} \times \mathbf{v})$
3. $\mathbf{u} \times \mathbf{u} = \mathbf{0}$
4. $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$

Theorem 2

Let $\mathbf{u}$, and $\mathbf{v}$, be vectors.

1. $\mathbf{u} \times \mathbf{v}$ is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$
2. $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin \theta$, where $\theta = 0^\circ \leq \theta < 360^\circ$ is the angle between $\mathbf{u}$ and $\mathbf{v}$.

What is the implication of Theorem 2.2?

Proof Theorem 2.1

<u>**Example 2**</u>

Find the area of the triangle whose vertices are A(1,0,3), B(1,3,4), and C(-2,5,1).

Theorem 3

Let $\mathbf{u} = (u_1, u_2, u_3)$, $\mathbf{v} = (v_1, v_2, v_3)$, and $\mathbf{w} = (w_1, w_2, w_3)$

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ represents the volume of the parallelepiped with edges $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

Proof

<u>**Example 3**</u>

Find the volume of the parallelepiped with edges $\overrightarrow{OA} = (1, 0, 6)$, $\overrightarrow{OB} = (-1, 4, 1)$, and $\overrightarrow{OC} = (-3, 2, 4)$