

Notes

Chapter 06: Inequalities in Geometry

Unit 1: Inequalities and Indirect Proof

Section 1: Inequalities

on your desk

Properties of Inequalities

1. If $a > b$ and $c \geq d$, then $a + c > b + d$.

6.1

2. If $a > b$ and $c > 0$, then $ac > bc$ and $a/c > b/c$.

6.2

3. If $a > b$ and $c > 0$, then $ac < bc$ and $a/c < b/c$.

6.3

4. If $a > b$ and $b > c$, then $a > c$.

5. If $a = b + c$ and $c > 0$, then $a > b$.

6.4

6.5

Can you state the exterior angle theorem?

Theorem 6-1 (The Exterior Angle Inequality Theorem)

The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.

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Exercise

Complete with $<$, $=$, or $>$.

6.1

1. Given: $RS < ST$; $ST < RT$

6.2

Conclusion: RS _____ RT

6.3

2. Given: $m\angle PQU = m\angle PQT + m\angle TQU$

6.4

Conclusion: $m\angle PQU$ _____ $m\angle PQT$, $m\angle PQU$ _____ $m\angle TQU$

6.5

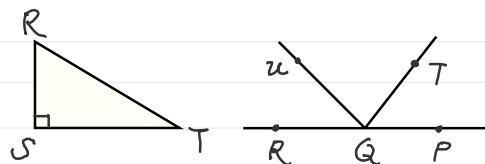
Classify each conditional as true or false.

3. If $XY = YZ + 5$, then $XY > YZ$.

4. If $m\angle A = m\angle B + m\angle C$, then $m\angle B > m\angle C$.

5. If $m\angle H = m\angle J + m\angle K$, then $m\angle K > m\angle H$.

6. If $10 = y + 2$, then $y > 10$.



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Write a two-column proof.

7. Given: $m\angle R = m\angle TUS$

Prove: $m\angle TUR > m\angle R$

6.1

6.2

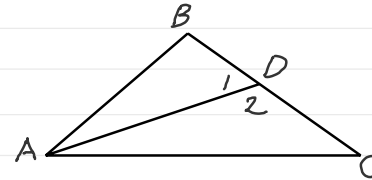
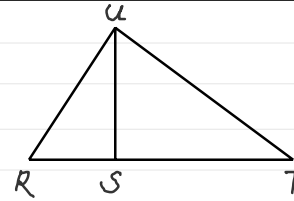
6.3

6.4

6.5

8. Given: $\angle 1$ is an exterior angle of $\triangle ACD$; $m\angle B > m\angle 1$

Prove: $m\angle B > m\angle C$



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Write a two-column proof.

7. Given: $m\angle R = m\angle TUS$

Prove: $m\angle TUR > m\angle R$

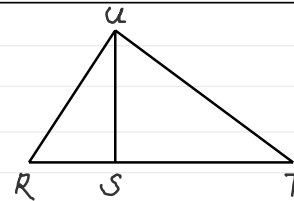
6.1

6.2

6.3

6.4

6.5



Statements

Reasons

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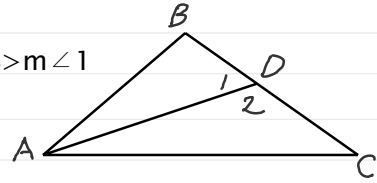
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8. Given: $\angle 1$ is an exterior angle of $\triangle ACD$; $m\angle B > m\angle 1$

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6.1

6.2

6.3

Statements

Reasons

6.4

6.5

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Summary of Related If-Then Statements

Direct statement: If p, then q. ($p \rightarrow q$)

6.1

Contrapositive: If not q, then not p. ($\sim q \rightarrow \sim p$)

6.2

Converse: If q, then p. ($q \rightarrow p$)

6.3

Inverse: If not p, then not q. ($\sim p \rightarrow \sim q$)

6.4

6.5

A statement and its contrapositive are **logically equivalent**.

A statement is *not* logically equivalent to its converse or to its inverse.

A converse and inverse are contrapositive of each other, which means that they are logical equivalents.

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Section 2: Inverses and Contrapostives

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Example

1. Direct statement: If I live in San Jose, then I live in California.

6.1

6.2

Contrapostive:

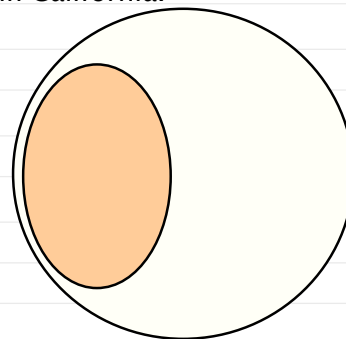
6.3

6.4

Converse:

6.5

Inverse:



Which statement are true?

2. Write "All squares are rhombuses" in if-then form.

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Example

Write (a) the inverse, (b) the contrapostive, and (c) the converse of each true conditional.

6.1

6.2

3. If a parallelogram is a square, then it is a rectangle.

6.3

6.4

6.5

4. If it is snowing, then the game is canceled.

5. If $2x+1 > 15$, then $x > 7$.

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How to Write an Indirect Proof

1. Assume temporarily that the conclusion is not true.
2. Reason logically until you reach a contradiction of a known fact.
3. Point out that the temporary assumption must be false, and that the conclusion must then be true.

6.1**6.2****6.3****6.4**

Indirect Proof is sometimes known as Proof by Contradiction.

6.5**Example**

Write an indirect proof in paragraph form.

1. Given: $m\angle X \neq m\angle Y$
Prove: $\angle X$ and $\angle Y$ are not both right angles
2. Given: $2r+3 \neq 17$
Prove: $r \neq 7$

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Example

Write an indirect proof in paragraph form.

1. Given: $m\angle X \neq m\angle Y$
Prove: $\angle X$ and $\angle Y$ are not both right angles

6.1**6.2****6.3****6.4****6.5**

2. Given: $2r+3 \neq 17$
Prove: $r \neq 7$

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Practice

Write an indirect proof in paragraph form.

6.11. Given: Tim drove 105 miles to his friend's house in $1\frac{1}{2}$ hours.**6.2**

Prove: Tim exceeded the 55mph speed limit while driving.

6.3**6.4****6.5**2. Given: $\square XYZW$; $m\angle X = 80$ Prove: $\square XYZW$ is not a rectangle

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Quick Quiz

Classify each conditional as true or false.

6.11. If $a \leq b$, then $b \geq a$.**6.2****6.3**2. If $r = s$ and $s < t$, then $r < t$.**6.4**3. If $m > n$ and $n < r$, then $m > r$.**6.5**

4. Given: If you live in Denver, then you live in Colorado.

a) Write the inverse of the given statement. Is it true?

b) Write the contrapositive of the given statement. Is it true?

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Quick Quiz**6.1**

5. Choose the letter of the statement that is locally equivalent to “If you will stay, then Kim will go.”

6.2**6.3**

A. If you won't stay, then Kim will go.

B. If you won't go, then you will stay.

6.4

C. If you won't stay, then Kim won't go.

6.5

D. If Kim won't go, then you won't stay.

6. Given: All square are rectangles.

What can you conclude from each additional statement? If no conclusion is possible, write *no conclusion*.

a. RSTU is not a rectangle

b. ABCD is not a square.

c. HIJK is a rectangle.

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6.1**6.2****6.3****6.4****6.5**

Given the above information, what can you say about the $\angle B$ and $\angle S$?

Conclusion 1:

Conclusion 2:

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Theorem 6-2

If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side.

6.1

6.2

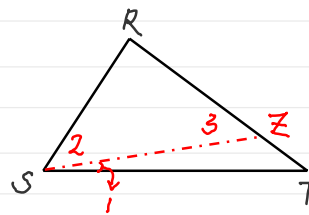
Given: $\triangle RST$; $RT > RS$

6.3

Prove: $m\angle RST > m\angle T$

6.4

6.5



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Theorem 6-3

If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.

6.1

6.2

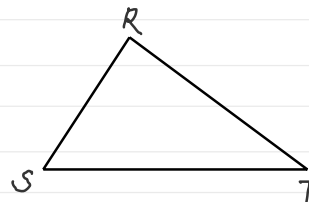
Given: $\triangle RST$; $m\angle S > m\angle T$

6.3

Prove: $RT > RS$

6.4

6.5



Corollary 1

The perpendicular segment from a point to a line is the shortest segment from the point to the line.

Corollary 2

The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

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Theorem 6-4 (Triangle Inequality)

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

6.1

6.2

Given: $\triangle ABC$

6.3

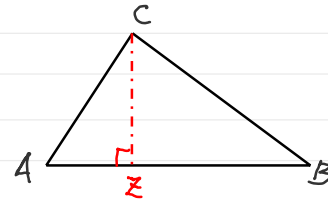
Prove: (1) $AB+BC>AC$

(2) $AB+AC>BC$

6.4

(3) $BC+AC>AB$

6.5



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How do you tell if a triangle is right, obtuse, or acute?

Hint 1: Most eloquent theorem in Geometry.

6.1

6.2

$m\angle C$ is right: $c^2=a^2+b^2$

6.3

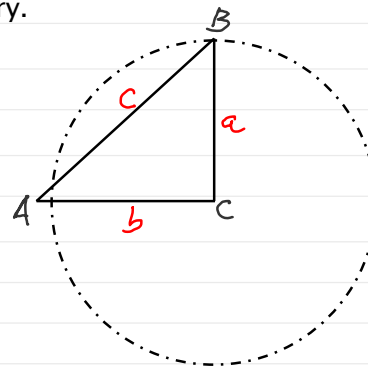
$m\angle C$ is obtuse: $c^2>a^2+b^2$

$m\angle C$ is acute: $c^2<a^2+b^2$

6.4

NOTE: c is the longest side.

6.5



Example

Determine if the following set of number form a triangle, then decide if the triangle is right, obtuse, or acute.

(a) 3,4,5

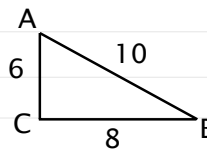
(b) 3,4,6

(c) 5,6,7

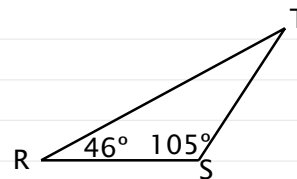
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Examples

1. Name the largest angle and the smallest angle of the triangle ABC.

**6.1****6.2****6.3****6.4****6.5**

2. Name the longest side and the shortest side of the triangle RST.



3. Complete with $<$, $>$, or $=$.

Given: $\triangle ABC$ is a right triangle with $m\angle C = 90$.

Conclusions: $m\angle C$ _____ $m\angle A$; $m\angle C$ _____ $m\angle A + m\angle B$; AC _____ AB

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on your desk

Examples

4. The lengths of two sides of a triangle are 8 and 13. Then, the length of the third side must be greater than _____ but less than _____.

6.1**6.2****6.3****6.4****6.5**

5. Is it possible for a triangle to have sides with the lengths as indicated? If yes, determine if the triangle is right, obtuse, or acute.

(a) 6, 8, 10 (b) 3, 4, 8 (c) 2.5, 4.1, 5 (d) 6, 6, 5

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Section 5: Inequality for Two Triangles

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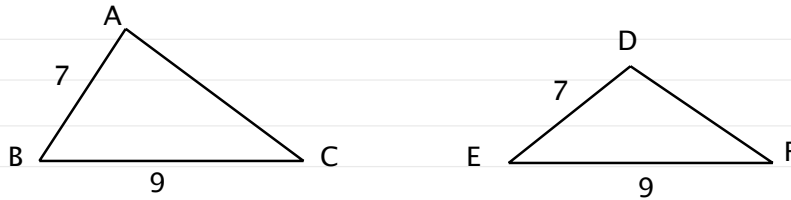
6.1

6.2

6.3

6.4

6.5



Given the above information, what can you say about the AC and DF?

Conclusion 1:

Conclusion 2:

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Section 5: Inequality for Two Triangles

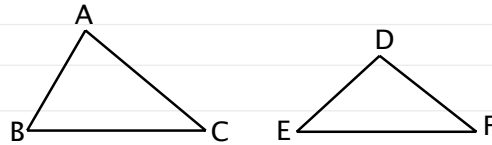
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Theorem 6-5 (SAS Inequality Theorem)

If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is larger than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle.

Given: $AB=DE$; $BC=EF$; $m\angle B > m\angle E$

Prove: $AC > DF$



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Section 5: Inequality for Two Triangles

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Theorem 6-6 (SSS Inequality Theorem)

If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second triangle.

6.1

6.2

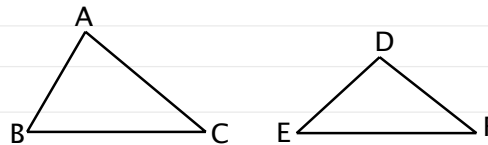
6.3

Given: $AB=DE$; $BC=EF$; $AC>DF$

6.4

Prove: $m\angle B>m\angle E$

6.5



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Examples

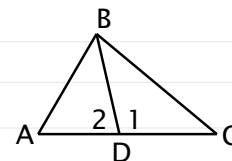
1. Given: D is the midpoint of segment AC; $m\angle 1 < m\angle 2$

What can you deduce?

6.1

6.2

6.3



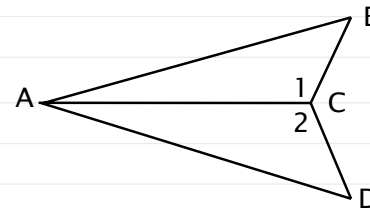
2. Write a two-column proof.

Given: $BC=DC$; $AB>AD$

6.4

Prove: $m\angle 1 > m\angle 2$

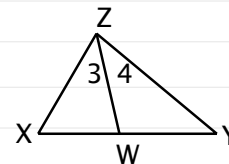
6.5



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Practice

1. What can you deduce? Name the theorem that supports your answer.



6.1

6.2

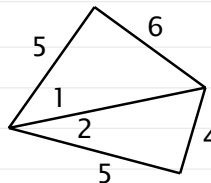
6.3

Given: $XZ=YS$; $m\angle 3 < m\angle 4$

6.4

6.5

Complete with $<$, $>$, or $=$.



2. $m\angle 1$ ____ $m\angle 2$

3. $m\angle SRT$ ____ $m\angle URV$

