Notes

Chapter 04: Congruent Triangles
Unit 1: Corresponding Parts in a Congruence
Section 1: Congruent Figures

on your desk

Whenever two figures have the same size and shape, they are called **congruent.**

4.1
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4.7

Triangles ABC and DEF are congruent. You can match up vertices like

A★D B★E C★F

This means that:

Corresponding Angles	*Corresponding Sides*
∠A ★_____	segment AB ★_____
∠ B★ _____	segment BC ★_____
∠ C★ _____	segment AC ★_____

Corresponding parts of a congruent triangles are congruent. (CPCTC)

Notes

Chapter 04: Congruent Triangles
Unit 1: Corresponding Parts in a Congruence
Section 1: Congruent Figures

on your desk

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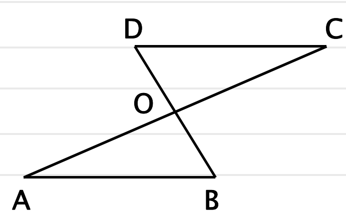
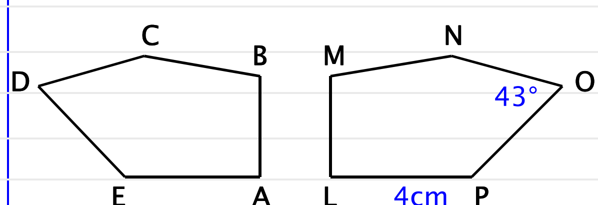
<u>Example 1</u> *Two triangles are congruent.*



a. △ABO≅_________
b. ∠B≅_________
c. ∠A≅ _________
d. segment AO≅ _________
e. segment AB≅ _________

<u>Example 2</u> *Two triangles are congruent.*



a. B corresponds to ______
b. m∠D≅_________
c. ∠D≅ _________
d. segment AE≅ _________
e. AE= _________

on your desk
4.1
4.2
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4.6
4.7

Postulate 12 (SSS Postulate)

If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

Postulate 13 (SAS Postulate)

If two sides and an included angle of one triangle are congruent to two sides and an included angle of another triangle, then the triangles are congruent.

Postulate 14 (ASA Postulate)

If two angles and an included side of one triangle are congruent to two angles and an included side of another triangle, then the triangles are congruent.

on your desk
4.1
4.2
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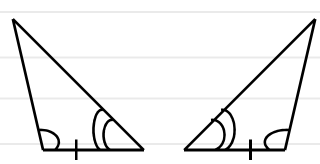
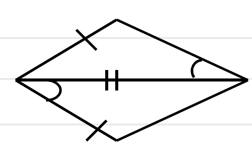
4.7

<u>Example 1</u> *Which of the three postulates do you use?*

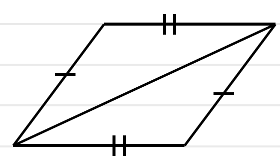
a.



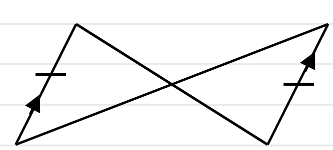
d.



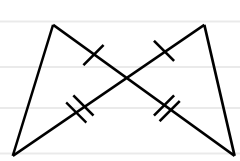
b.



e.



c.



Notes

Chapter 04: Congruent Triangles
Unit 1: Corresponding Parts in a Congruence
Section 3: Using Congruent Triangles

on your desk

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4.3
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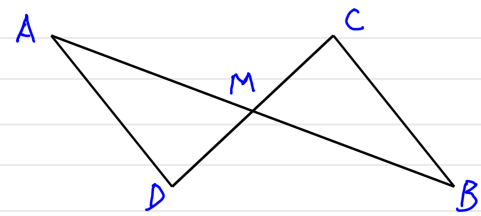
4.7

Example 1

Given: segment AB and segment CD bisect each other at M

Prove: segment AD || segment BC



Statements	*Reasons*

Notes

Chapter 04: Congruent Triangles
Unit 1: Corresponding Parts in a Congruence
Section 3: Using Congruent Triangles

on your desk

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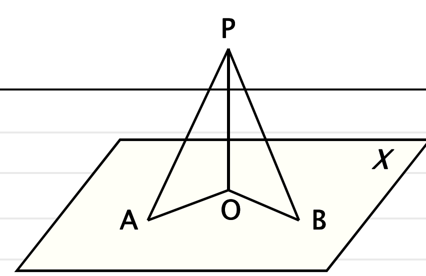
4.6
4.7

Example 2

Given: segment PO ⊥ plane X; segment PO bisects ∠APB

Prove: segment AD || segment BC



Statements	*Reasons*

Notes

Chapter 04: Congruent Triangles
Unit 1: Corresponding Parts in a Congruence
Section 3: Using Congruent Triangles

on your desk

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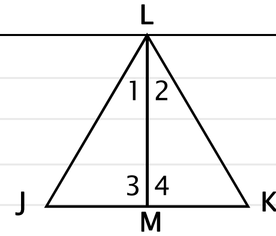
4.6
4.7

Example 3

Given: $m\angle 1 = m\angle 2$; $m\angle 3 = m\angle 4$

Prove: M is the midpoint of segment JK



Statements	*Reasons*

Notes

Chapter 04: Congruent Triangles
Unit 2: Some Theorems Based on Congruent Triangles
Section 4: The Isosceles Triangle

on your desk

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4.2
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4.7

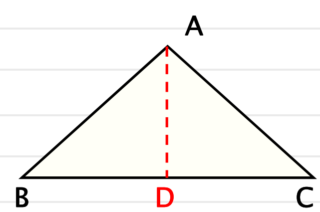
Theorem 4-1 (The Isosceles Triangle Thm)

If two sides of a triangles are congruent, then the angles opposite those sides are congruent.

Proof

Given: segment AB ≅ segment AC

Prove: $\angle B \cong \angle C$



Statements	*Reasons*

Notes

Chapter 04: Congruent Triangles
Unit 2: Some Theorems Based on Congruent Triangles
Section 4: The Isosceles Triangle

on your desk

4.1 4.2 4.3 4.4 4.5 4.6 4.7

Theorem 4–1 (The Isosceles Triangle Thm)

If two sides of a triangles are congruent, then the angles opposite those sides are congruent.

Corollary 1

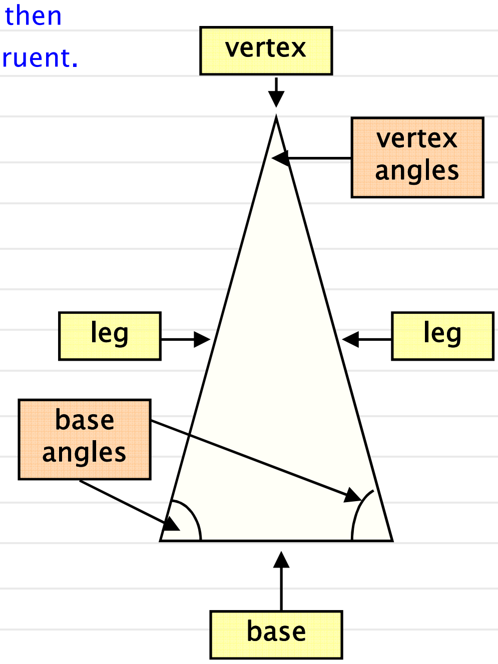
An equilateral triangle is also equiangular.

Corollary 2

An equilateral triangle has three 60° angles.

Corollary 3

The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.



Notes

Chapter 04: Congruent Triangles
Unit 2: Some Theorems Based on Congruent Triangles
Section 4: The Isosceles Triangle

How would you prove this?

on your desk

4.1 4.2 4.3 4.4 4.5 4.6 4.7

Theorem 4–2 (The Converse of Isosceles Triangle Thm)

If two angles of a triangles are congruent, then the sides opposite those angles are congruent.

Corollary 4

An equiangular triangle is also equilateral.

on your desk
4.1
4.2
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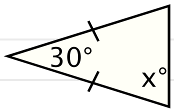
4.7

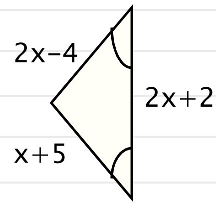
Guided Practice

Find the value of x.

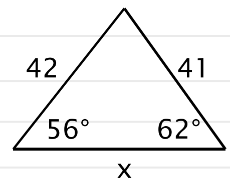
(1)



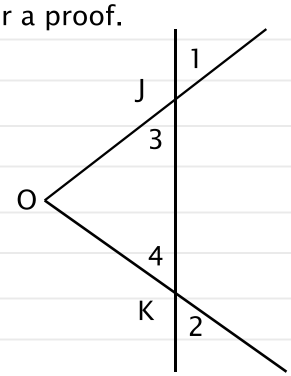
(2)



(3)



(4) Place the statements in an appropriate order for a proof.



Given: $\angle 1 \cong \angle 2$

Prove: segment OK ≅ segment OJ

(a) $\angle 3 \cong \angle 4$

(b) $\angle 2 \cong \angle 4$; $\angle 3 \cong \angle 1$

(c) segment OK ≅ segment OJ

(d) $\angle 1 \cong \angle 2$

on your desk
4.1
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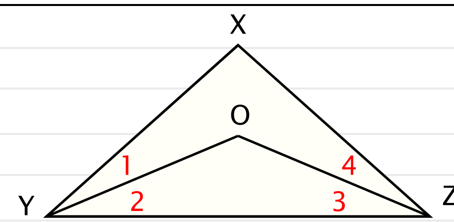
Guided Practice

(5) Do the proof of the following



Given: segment XZ ≅ segment XY;
ray YO bisects $\angle XYZ$;
ray ZO bisects $\angle XZY$

Prove: segment YO ≅ segment ZO

Statements	*Reasons*

Notes

Chapter 04: Congruent Triangles
Unit 2: Some Theorems Based on Congruent Triangles
Section 5: Other Methods of Proving Triangles Congruent

on your desk
4.1
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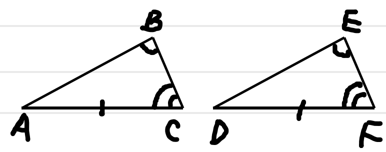
4.6
4.7

Theorem 4-3 (AAS Thm)

If two sides and a non-included angle of one triangle are congruent to the corresponding parts of another triangle, then the triangles are congruent.



Given: $\triangle ABC$ and $\triangle DEF$; $\angle B \cong \angle E$;
$\angle C \cong \angle F$; segment AC ≅ segment DF

Prove: $\triangle ABC \cong \triangle DEF$

Statements	*Reasons*

Notes

Chapter 04: Congruent Triangles
Unit 2: Some Theorems Based on Congruent Triangles
Section 5: Other Methods of Proving Triangles Congruent

on your desk
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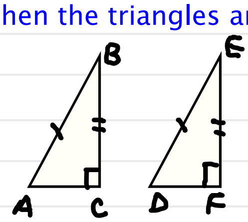
Theorem 4-4 (HL Thm)

If the hypotenuse and a leg of one right triangle are congruent to the corresponding parts of another right triangle, then the triangles are congruent.



Given: $\triangle ABC$ and $\triangle DEF$; $\overline{AB} \cong \overline{DE}$; $\overline{BC} \cong \overline{EF}$
$\angle C$ and $\angle F$ are right $\angle$s

Prove: $\triangle ABC \cong \triangle DEF$

Statements	*Reasons*

Chapter 04: Congruent Triangles

Unit 2: Some Theorems Based on Congruent Triangles

Section 6: Using More than One Pair of Congruent Triangle

on your desk
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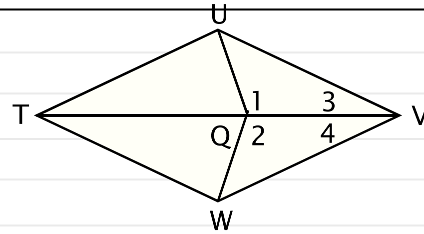
4.7

Example

Given: $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$

Prove: segment TU ≅ segment TW



Statements	Reasons

Chapter 04: Congruent Triangles

Unit 2: Some Theorems Based on Congruent Triangles

Section 6: Using More than One Pair of Congruent Triangle

on your desk
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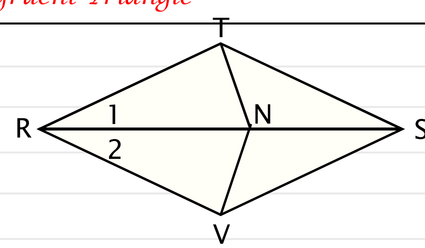
Example

Given: segment RT ≅ segment RV

segment NT ≅ segment NV

Prove: segment TS ≅ segment VS



Statements	Reasons

on your desk

4.1
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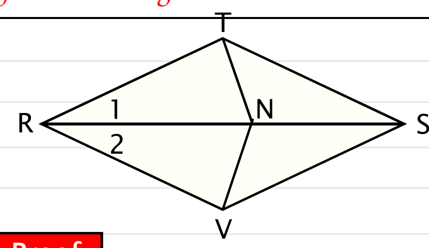
Example

Given: segment RT ≅ segment RV
segment NT ≅ segment NV

Prove: segment TS ≅ segment VS



Prove by Paragraph Proof

Given that segment RT ≅ segment RV, segment NT ≅ segment NV, and segment RN ≅ segment RN by reflexive property, $\triangle RNT \cong \triangle RNV$ by SSS.
We note that segment RS ≅ segment RS by reflexive property and $\angle 1 \cong \angle 2$ by CPCTC, so $\triangle RTS \cong \triangle RVS$ by SAS.
Therefore, segment TS ≅ segment VS by CPCTC.

on your desk

4.1
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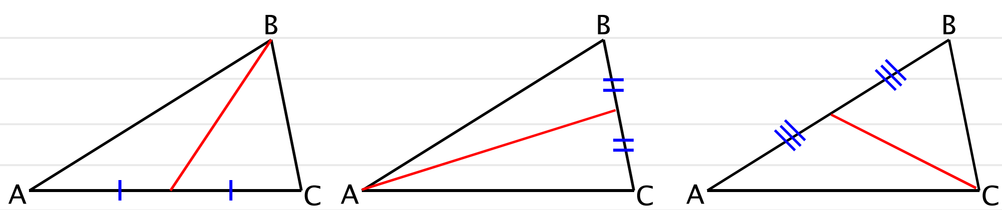
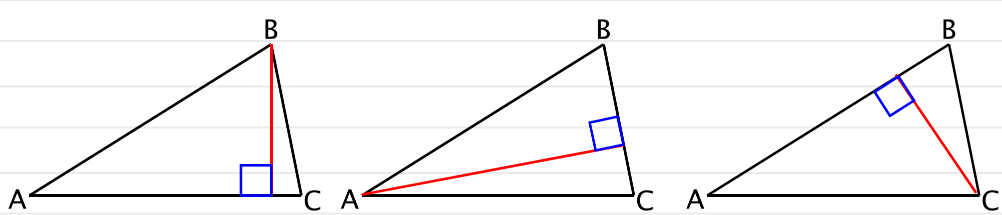
4.7

A **median** of a triangle is a segment from a vertex of a triangle to the midpoint of the opposite side.



An **altitude** of a triangle is a perpendicular segment from a vertex of a triangle to the line containing the opposite side.



on your desk

4.1
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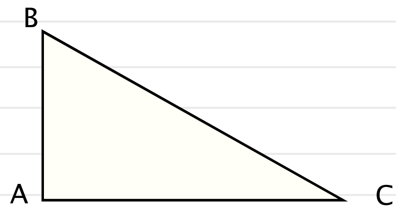
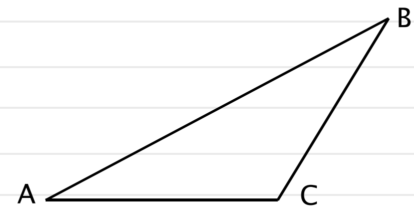
Draw three altitude on the following right triangle and obtuse triangle.





A **perpendicular bisector** of a segment is a line (or ray or segment) that is perpendicular to the segment at its midpoint.

on your desk

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4.2
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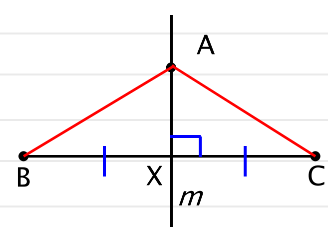
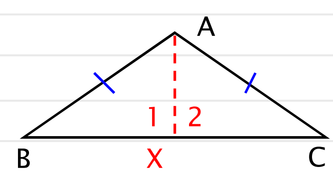
Theorem 4–5

If a point lies on the perpendicular bisector of a segment, then the point is equidistant from the endpoints of the segment.



Theorem 4–6

If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment.



Chapter 04: Congruent Triangles
Unit 2: Some Theorems Based on Congruent Triangles
Section 7: Medians, Altitudes, and Perpendicular Bisectors

on your desk
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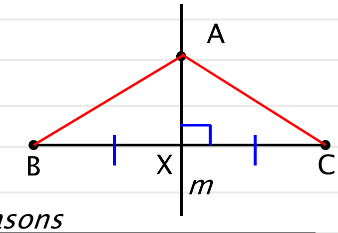
Theorem 4–5

If a point lies on the perpendicular bisector of a segment, then the point is equidistant from the endpoints of the segment.



Given: line m is the ⊥ bisector of segment BC;
A is on m

Prove: AB ≅ AC

Statements	*Reasons*

Chapter 04: Congruent Triangles
Unit 2: Some Theorems Based on Congruent Triangles
Section 7: Medians, Altitudes, and Perpendicular Bisectors

on your desk
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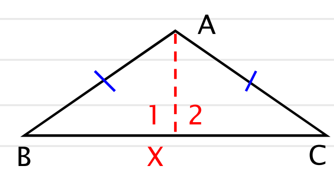
Theorem 4–6

If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment.



Given: AB=AC

Prove: A is on the ⊥ bisector of segment BC

Statements	*Reasons*

on your desk
4.1
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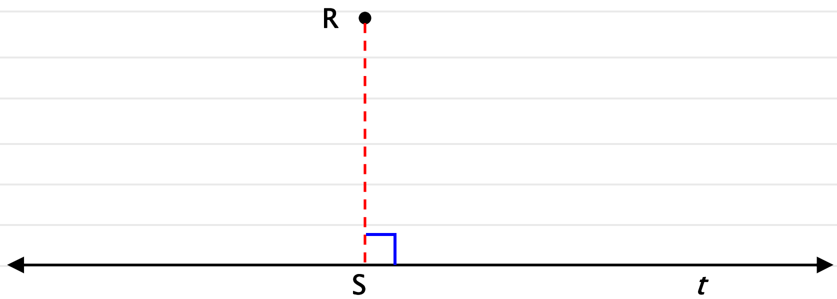
The **distance from a point to a line** (or plane) is defined to be the length of the perpendicular segment from the point to the line (or plane).



The length of segment RS, denoted RS, is the distance between the point P and the line *t*.

on your desk
4.1
4.2
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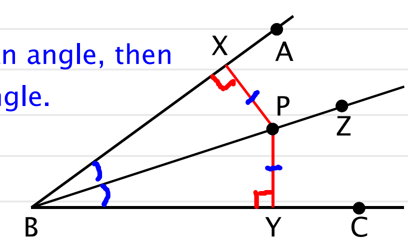
4.4
4.5
4.6
4.7

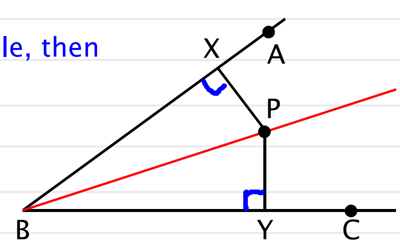
Theorem 4–7

If a point lies on the perpendicular bisector of an angle, then the point is equidistant from the sides of the angle.



Theorem 4–8

If a point is equidistant from the sides of an angle, then the point lies on the bisector of the angle.



Notes

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on your desk
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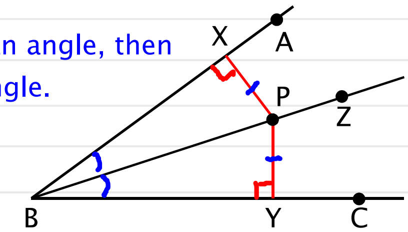
Theorem 4–7

If a point lies on the perpendicular bisector of an angle, then the point is equidistant from the sides of the angle.



Given: ray BZ bisects ∠ABC; P lies on ray BZ;
segment PX ⊥ ray BA;
segment PY ⊥ ray BC

Prove: PX=PY

Statements	*Reasons*

Notes

Chapter 04: Congruent Triangles
Unit 2: Some Theorems Based on Congruent Triangles
Section 7: Medians, Altitudes, and Perpendicular Bisectors

on your desk
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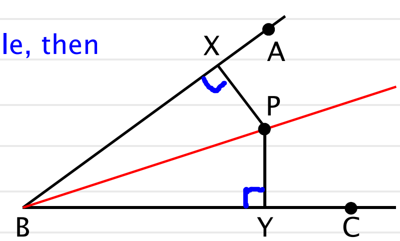
4.7

Theorem 4–8

If a point is equidistant from the sides of an angle, then the point lies on the bisector of the angle.



Given: segment PX ⊥ ray BA;
segment PY ⊥ ray BC; PX=PY

Prove: ray BP bisects ∠ABC

Statements	*Reasons*

on your desk
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Chalkboard Examples

Fill in the blank with always, sometimes, or never.

1. An altitude is ____________ perpendicular to the opposite side.
2. A median is ____________ perpendicular to the opposite side.
3. An altitude is ____________ an angle bisector.
4. An angle bisector is ____________ perpendicular to the opposite side.
5. A perpendicular bisector of a segment is ____________ equidistant form the endpoints of the segment.
6. Suppose ray OG bisects angle TOY. What can you deduce if you also know that:
   a. Point J lies on ray OG?
   b. A point K is such that the distance from K to ray OT is 13 cm and the distance from K to ray OY is 13 cm?

on your desk
4.1
4.2
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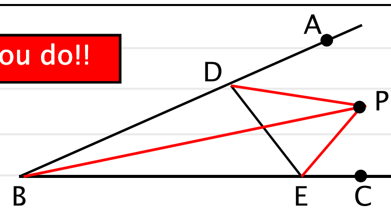
4.6
4.7

Examples

You do!!

Given: ray DP bisects ∠ADE;
ray EP bisects ∠DEC;
Prove: ray BP bisects ∠ABC



Statements	*Reasons*