Notes

Chapter 02: Deductive Reasoning

Unit 1: Using Deductive Reasoning

Section 1: If-Then Statements; Converse

on your desk

If-then statement are also known as **conditional statements** or simply **conditionals**.

2.1

2.2 **Examples**

2.3 "If it rains, then I use an umbrella."

"If cats meow, then dogs bark."

2.4 "If p, then q."

2.5 p: hypothesis (condition) q: conclusion

2.6

Converse statement reverses the hypothesis and the conclusion.

Statement:	If Joseph lives in Saratoga, then he lives in California.
Converse:	If _____, then _____.
Inverse:	If _____, then _____.
Contrapositive:	If _____, then _____.

Is the converse true in this case? Inverse? Contrapositive?

What can you say about the converse statement in general? Inverse? Contrapositive?

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Chapter 02: Deductive Reasoning

Unit 1: Using Deductive Reasoning

Section 1: If-Then Statements; Converse

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General Form	Example
If p, then q.	If $x^2=25$, then $x<10$.
2.1 p implies q.	$x^2=25$ implies $x<10$.
2.2 p only if q.	$x^2=25$ only if $x<10$.
2.3 q if p.	$x<10$ if $x^2=25$.

2.4 If a conditional and its converse are both true, they can be combined into a

2.5 single statement using "if and only if." This combined statement is called

2.6 **biconditional**.

Conditional:	If p, then q. $(p \rightarrow q)$
Converse:	If q, then p. $(p \leftarrow q$ or $q \rightarrow p)$
Inverse:	If not p, then not q. $(\sim p \rightarrow \sim q)$
Contrapositive:	If not q, then not p. $(\sim q \rightarrow \sim p)$
Bicondiational:	p if and only if q. $(p \leftrightarrow q)$

on your desk

Guided Practice

Write the hypothesis and the conclusion of each conditional.

2.1 1. VW=XY implies segment VW ≅ segment XY.

2.2

2.3

2. I run, if you run.

2.4

2.5

2.6 3. n>8 only if n is greater than 7.

4. If a=b, then a+c=b+c.

5. r-t=s-t only if r=s.

on your desk

Guided Practice

Provide a counterexample to show that each statement is false.

2.1 6. If segment AB ≅ segment BC, then B is the midpoint of segment AC.

2.2

2.3

7. If a line lies in a vertical plane, then the line is vertical.

2.4

2.5

2.6 8. If a number is divisible by 4, then it is divisible by 6.

9. If $x^2=49$, the x=7.

Notes

Chapter 02: Deductive Reasoning

Unit 1: Using Deductive Reasoning
Section 1: If-Then Statements; Converse

You do!!!!

on your desk

Guided Practice

Write the converse, inverse, and contrapositive of each conditional. Is the converse true?

2.1 2.2 2.3 2.4 2.5 2.6

10. If today is Monday, then tomorrow is Tuesday.
    - Converse
    - Inverse
    - Contrapositve

11. If a number is divisible by 6, then it is divisible by 3.
    - Converse
    - Inverse
    - Contrapositve

Notes

Chapter 02: Deductive Reasoning

Unit 1: Using Deductive Reasoning
Section 2: Properties from Algebra

We do!!!!

on your desk

2.1 2.2 2.3 2.4 2.5 2.6

Algebra Definitions	Definitions
Addition Property of Equality	
Subtraction Prop. of =	
Multiplication Prop. of =	
Division Prop. of =	
Substitution Prop. of =	
Reflexive Prop. of =	
Symmetric Prop. of =	
Transitive Prop. of =	
Reflexive Prop. of Congruence	
Symmetric Prop. of ≅	
Transitive Prop. of ≅	
Distributive Property	
Prop. of ≅	

We do!!!!

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Example 1

Justify each step in solving the equation $5x-3=\frac{3x}{2}$.

2.1 2.2 2.3 2.4 2.5 2.6

	STEP	Reasons
1.	$5x-3=\frac{3x}{2}$	1.
2.	$10x-6=3x$	2.
3.	$7x-6=0$	3.
4.	$7x=6$	4.
5.	$x=6/7$	5.

We do!!!!

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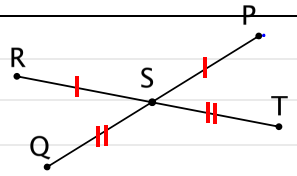
Example 2

Given: $\overline{RT}$ and $\overline{PQ}$ intersecting at S so that
$RS=PS$ and $ST=SQ$

Prove: $RT=PQ$



2.1 2.2 2.3 2.4 2.5 2.6

Statements	*Reasons*

Notes

Chapter 02: Deductive Reasoning

Unit 1: Using Deductive Reasoning

Section 2: Properties from Algebra

We do!!!!

on your desk

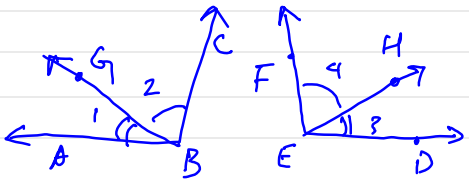
Example 3

Given: $m\angle 1 = m\angle 3$;

2.1 $m\angle 2 = m\angle 4$

2.2 Prove: $m\angle ABC = m\angle DEF$

2.3



2.4

2.5 *Statements* | *Reasons*

2.6

Notes

Chapter 02: Deductive Reasoning

Unit 1: Using Deductive Reasoning

Section 2: Properties from Algebra

We do!!!!

on your desk

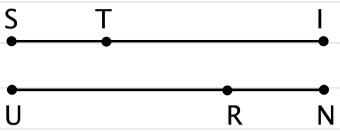
Example 4

Given: ST=RN; IT=RU

2.1 Prove: SI=UN

2.2

2.3



2.4

2.5 *Statements* | *Reasons*

2.6

We do!!!!

on your desk

Theorem 2-1 (Midpoint Theorem)

If M is the midpoint of segment AB, then $AM = \frac{1}{2}AB$ and $MB = \frac{1}{2}AB$.

2.1

2.2 Given: M is the midpoint of segment AB

2.3 Prove: $AM = \frac{1}{2}AB$ and $MB = \frac{1}{2}AB$

2.4 **Proof of Midpoint Theorem:**

2.5

Statements	*Reasons*

2.6

on your desk

Theorem 2-2 (Angle Bisector Theorem)

If ray BX is the bisector of angle ABC, then

2.1 $m\angle ABX = \frac{1}{2}m\angle ABD$ and $m\angle XBC = \frac{1}{2}m\angle ABC$.

2.2 Given: ray BX is the bisector of $\angle ABC$

2.3 Prove: $m\angle ABX = \frac{1}{2}m\angle ABD$ and $m\angle XBC = \frac{1}{2}m\angle ABC$

2.4 **Proof of Angle Bisector Theorem:**

2.5

Statements	*Reasons*

2.6

Notes

Chapter 02: Deductive Reasoning

Unit 1: Using Deductive Reasoning

Section 3: Proving Theorems

You do!!!!

on your desk

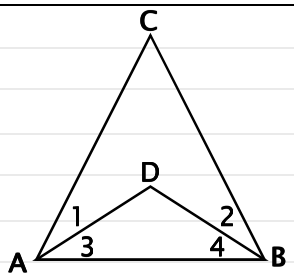
Chalkboard Example 1

Given: $m\angle 1 = m\angle 2$; ray AD bisects $\angle CAB$;

2.1 ray BD bisects $\angle CBA$

2.2 Prove: $m\angle 3 = m\angle 4$

2.3



2.4

2.5

2.6

Chalkboard Example 2

Given: WX=YZ;

Y is the midpoint of segment XZ

Prove: WX=XY

Notes

Chapter 02: Deductive Reasoning

Unit 1: Using Deductive Reasoning

Section 3: Proving Theorems

We do!!!!

on your desk

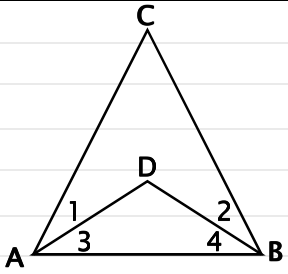
Chalkboard Example 1

Given: $m\angle 1 = m\angle 2$; ray AD bisects $\angle CAB$;

2.1 ray BD bisects $\angle CBA$

2.2 Prove: $m\angle 3 = m\angle 4$

2.3



2.4 *Statements* *Reasons*

2.5

2.6

Notes

Chapter 02: Deductive Reasoning

Unit 1: Using Deductive Reasoning

Section 3: Proving Theorems

We do!!!!

on your desk

Chalkboard Example 2

Given: WX=YZ;

2.1 Y is the midpoint of segment XZ

2.2 Prove: WX=XY

2.3

2.4

Statements	*Reasons*
2.5	
2.6	

Notes

Chapter 02: Deductive Reasoning

Unit 2: Theorems about Angles and Perpendicular Lines

Section 4: Special Pairs of Angles

on your desk

Complementary angles (comp. ∠s)

are two angles whose _____of angles is _____.

2.1

2.2

2.3

Supplementary angles (supp. ∠s)

2.4 are two angles whose _____ of angles is _____.

2.5

2.6

Vertical angles (vert. ∠s)

are two angles such that the sides of one angle are opposite

_____ to the sides of the other angle.

Notes

Chapter 02: Deductive Reasoning

Unit 2: Theorems about Angles and Perpendicular Lines

Section 4: Special Pairs of Angles

We do!!!!

on your desk

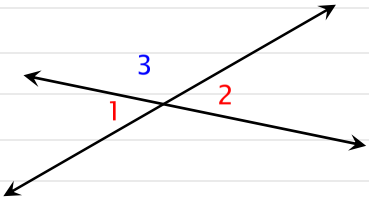
Theorem 2-3

Vertical angles are congruent.

2.1

2.2 Given: ∠1 and ∠2 are vertical angles

2.3 Prove: ∠1 ≅ ∠2



2.4 **Proof:**

2.5

Statements	*Reasons*

2.6

Notes

Chapter 02: Deductive Reasoning

Unit 2: Theorems about Angles and Perpendicular Lines

Section 5: Perpendicular Lines

on your desk

Definition

Two lines are perpendicular if and only if they intersect at _____

2.1 angles.

2.2 **Theorem 2-4**

2.3 If two lines are perpendicular, then they form _____ adjacent angles.

Theorem 2-5

2.4 If two lines form congruent adjacent angles, then the lines are

2.5 _____.

2.6

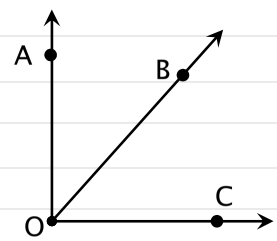
Theorem 2-6

If the exterior sides of two adjacent acute angles are perpendicular, then the angles are _____.

Given: ray OA ⊥ ray OC

Prove: ∠AOB and ∠BOC are comp. ∠s

Please prove the theorem 2.6



Notes

Chapter 02: Deductive Reasoning

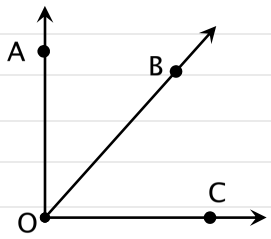
Unit 2: Theorems about Angles and Perpendicular Lines
Section 5: Perpendicular Lines

We do!!!!

on your desk

Theorem 2-6

If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.



Given: ray OA ⊥ ray OC

Prove: ∠AOB and ∠BOC are comp. ∠s

Statements	*Reasons*

Notes

Chapter 02: Deductive Reasoning

Unit 1: Using Deductive Reasoning
Section 1: If-Then Statements; Converse

You do!!!!

on your desk

Guided Practice

Complete with always, sometimes, or never.

1. Perpendicular lines _____ lie in the same plane.
2. Two lines are perpendicular if and only if they _____form congruent adjacent angles.
3. Perpendicular lines _____ form 60° angles.
4. If the exterior sides of two adjacent angles are perpendicular, then the angles are _____ supplementary.
5. If a pair of vertical angles are supplementary, the lines forming the angles are _____ perpendicular.

Notes

Chapter 02: Deductive Reasoning

Unit 2: Theorems about Angles and Perpendicular Lines
Section 5: Perpendicular Lines

You do!!!!

on your desk

Chalkboard Example

line AB ⊥ line CD. Use the diagram to classify each statement as true or false.

2.1
2.2
2.3

1. segment AB ⊥ segment EF

2.4

2. ∠ CGB is a right angle.

2.5
2.6

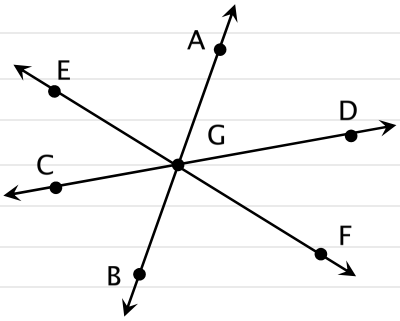
3. ∠CGA is a right angle.

4. m∠DGB=90.

5. ∠EGC and ∠EGA are complementary

6. ∠DGF is complementary to ∠DGA

7. ∠EGA is complementary to ∠DGF.



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Chapter 02: Deductive Reasoning

Unit 2: Theorems about Angles and Perpendicular Lines
Section 6: Planning a Proof

on your desk

A proof consists of five parts:

- *Statement* of the theorem
- A diagram that illustrates the *given information*
- A list, in terms of the figure, of what is *given*.
- A list, in terms of the figure, of what you are to *prove*
- A series of *statements and reasons* that lead from the given information to the statement that is to be proved

2.1
2.2
2.3
2.4
2.5
2.6

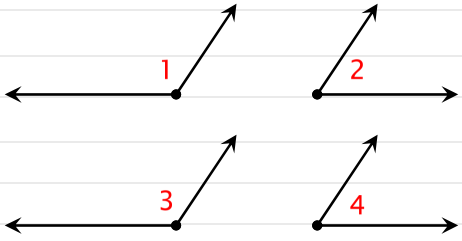
Theorem 2-7

If two angles are supplement of congruent angles (or of the same angle), then the two angles are congruent.

Given: ∠1 and ∠2 are supp.;
∠3 and ∠4 are supp.;
∠2≅∠4

Prove: ∠1≅∠3

How do we begin??



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Chapter 02: Deductive Reasoning

Unit 2: Theorems about Angles and Perpendicular Lines

Section 6: Planning a Proof

We do!!!!

on your desk

Theorem 2-7

If two angles are supplement of congruent angles (or of the same angle), then the two angles are congruent.

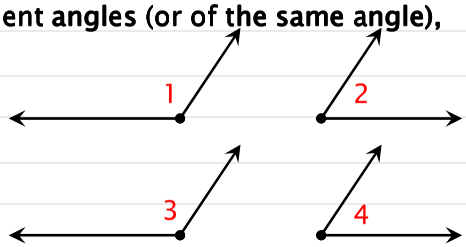
2.1

2.2 Given: ∠1 and ∠2 are supp.;

2.3 ∠3 and ∠4 are supp.;

∠2≅∠4

2.4 Prove: ∠1≅∠3



2.5

Statements	*Reasons*

2.6

Notes

Chapter 02: Deductive Reasoning

Unit 2: Theorems about Angles and Perpendicular Lines

Section 6: Planning a Proof

You do!!!!

on your desk

Theorem 2-8

If two angles are complement of congruent angles (or of the same angle), then the two angles are congruent.

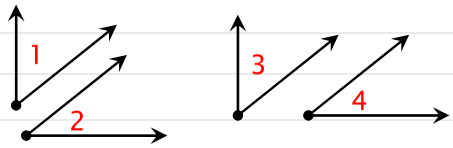
2.1

2.2 Given: ∠1 and ∠2 are comp.;

2.3 ∠3 and ∠4 are comp.;

∠2≅∠4

2.4 Prove: ∠1≅∠3



2.5

Statements	*Reasons*

2.6

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Chapter 02: Deductive Reasoning

Unit 2: Theorems about Angles and Perpendicular Lines

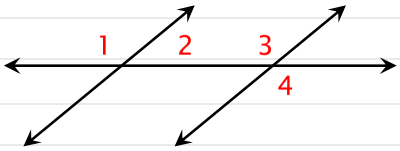
Section 6: Planning a Proof

You do!!!!

on your desk

Chalkboard Example

Explain your plan for proof.



2.1

2.2 **Question 1**

2.3 Given: $\angle 2$ and $\angle 3$ are supp.

Prove: $m\angle 1 = m\angle 3$

2.4

2.5

2.6

Notes

Chapter 02: Deductive Reasoning

Unit 2: Theorems about Angles and Perpendicular Lines

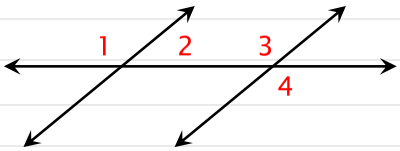
Section 6: Planning a Proof

You do!!!!

on your desk

Chalkboard Example

Explain your plan for proof.



2.1

2.2 **Question 2**

2.3 Given: $m\angle 1 = m\angle 4$

Prove: $\angle 4$ is supp. to $\angle 2$

2.4

2.5

2.6

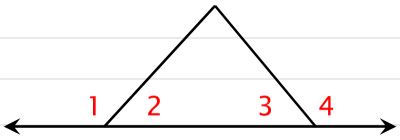
You do!!!!

on your desk

Guided Practice

Question 1

2.1 *Explain your plan for proof.*



2.2 a. Name a supplement of ∠2.

2.3

2.4 b. Name a supplement of ∠3.

2.5

2.6

c. What postulate or theorem, along with the definition of supplementary angles, justifies your answer to part (a) and (b)?

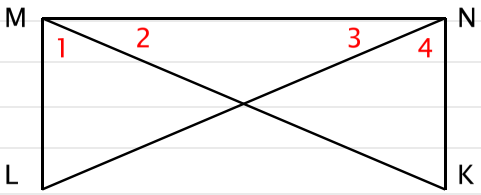
d. If ∠2≅∠3, write the theorem that allows you to conclude that 1∠≅∠4.

You do!!!!

on your desk

Guided Practice cont'd

Question 2



2.1 segment LM⊥segment MN,

2.2 and segment KN ⊥segment MN

2.3

a. Name a complement of ∠2.

2.4

2.5 b. Name a complement of ∠3.

2.6

c. What postulate or theorem justifies your answer to part (a) and (b)?

d. If ∠2≅∠3, write the theorem that allows you to conclude that 1∠≅∠4.